The Drude Model

Theory of metallic conduction. (1900)

→ Based on the kinetic theory

→ Consider a metallic element which has mass denisty ρ_m, atomic mass A.

\# of atoms per mole ~~mole~~ – 6.02×10^{23}

If Z is the number of valence electrons; then electrondensity in metallic element is given by

$$\boxed{n = 6.02 \times 10^{23} \frac{Z \rho_m}{A}}$$

Basic Assumptions —

(i) Free electron approximation: Between collisions the interactions of a given electron both with the others and with the ions is neglected.

(ii) Similar to kinetic theory, collisions in Drude model are instantaneous events that quickly changes the velocity of an electron.

(iii) $\tau \rightarrow$ mean free time. We shall assume that an electron experiences a collision with a probability time $\frac{1}{\tau}$.
$\tau = (t_1 + t_2 + \cdots t_n)/n$

(iv) Equilibrium assumption: Electrons are assumed to achieve thermal equilibrium with their surroundings through collisions.

DC electrical conductivity of a metal:

For isotropic situation, electric field $\vec{E}$ and current density $\vec{J}$ has relation $\vec{E} = \rho \vec{J}$, where, ρ is resistivity

~~$\vec{J} = \sigma \vec{E}$~~, $\vec{J}$ related to electron density n by $\vec{J} = -ne\vec{v}$, $\vec{v} \rightarrow$ velocity of e^-

This gives $\vec{J} = \left(\frac{ne^2\tau}{m}\right)\vec{E}$, or $\vec{J} = \sigma \vec{E}$, $\boxed{\sigma = \frac{ne^2\tau}{m}}$

AC electrical conductivity of a metal

$$\vec{E}(t) = Re\left(\vec{E}(\omega)\, e^{-i\omega t}\right) \quad \text{— (1)}$$

→ is time dependent electric field

Eqⁿ for momentum per electron is

$$\frac{d\vec{p}}{dt} = \frac{-\vec{p}}{\tau} - e\vec{E} \quad \text{— (3)}$$

we can take a steady state soln. of the form

$$\vec{p}(t) = Re\left(\vec{p}(\omega)\, e^{-i\omega t}\right) \quad \text{— (2)}$$

(1) & (2) in (3)

$$-i\omega \vec{p}(\omega) = \frac{-\vec{p}(\omega)}{\tau} - e\vec{E}(\omega)$$

$$\Rightarrow \vec{J} = -\frac{ne\vec{p}}{m}$$

so $$\vec{J}(t) = Re\left(\vec{J}(\omega)\, e^{-i\omega t}\right)$$

$$\vec{J}(\omega) = -\frac{ne\vec{p}(\omega)}{m} = \frac{\frac{ne^2}{m}\vec{E}(\omega)}{\frac{1}{\tau} - i\omega}$$

$$\Rightarrow \vec{J}(\omega) = \sigma(\omega)\vec{E}(\omega)$$

now $\sigma(\omega) = \dfrac{\sigma_0}{1 - i\omega\tau}$, $\sigma_0 = \dfrac{ne^2\tau}{m}$

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Now taking $\vec{J}(\vec{r},\omega) = \sigma(\omega)\vec{E}(\vec{r},\omega)$

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for $\lambda > l$

$\nabla\cdot E = 0$, $\nabla\cdot H = 0$ $\quad \nabla\times E = -\frac{1}{c}\frac{\partial \vec{H}}{\partial t}$

$$\nabla\times\vec{H} = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial\vec{E}}{\partial t}$$

$$\nabla\times(\nabla\times E) = -\frac{1}{c}\frac{\partial}{\partial t}(\nabla\times\vec{H})$$

$$-\nabla^2\vec{E} = \frac{i\omega}{c}\nabla\times\vec{H}$$

$$= \frac{i\omega}{c}\left(\frac{4\pi\sigma}{c}E - \frac{i\omega}{c}\vec{E}\right)$$

$$-\nabla^2\vec{E} = \frac{\omega^2}{c^2}\left(1 + \frac{4\pi i\sigma}{\omega}\right)\vec{E}$$

$$\nabla^2\vec{E} = \frac{\omega^2}{c^2}\varepsilon(\omega)\vec{E}$$

$$\varepsilon(\omega) = 1 + \frac{4\pi i\sigma}{\omega}$$

for $\omega\tau >> 1$

$$\boxed{\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}, \quad \omega_p^2 = \frac{4\pi n e^2}{m}}$$

$\omega < \omega_p \rightarrow \epsilon =$ real & $-$ve $\rightarrow$

soln. decay exponentially

no radiation can

propagate.

$\omega > \omega_p \rightarrow \epsilon = +$ve $\rightarrow$ oscillatory soln. $\rightarrow$ radiation propagate

transparent metal

Thermal conductivity of a metal

Success of Drude model → empirical law of Wiedemann & Franz law

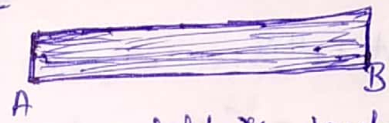
$$\frac{K}{\sigma} \propto T$$

K ← thermal conductivity

σ ← electrical conductivity

→ Bulk of thermal current is carried by thermal e^-'s (electrons)

Thermal conductivity



metal bar temp. varies slowly

If there were no source and sinks of heat at the ends of the bar to maintain the temp. gradient, the the hot end would cool and the cool end would warm, i.e., thermal energy would flow in a sense opposite to the temp. gradient

$\vec{j}^{\,q}$ → $\parallel^l$ to the direction of heat flow → thermal current density

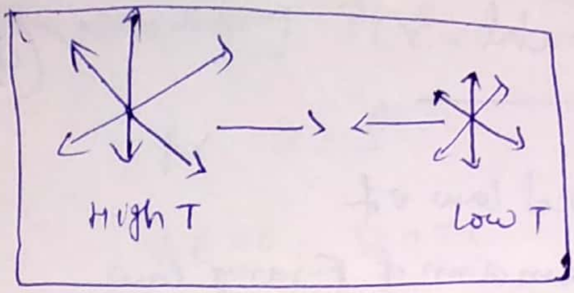
magnitude give thermal energy per unit area $\perp^{ar}$ to flow

for small temp. gradient

$$\vec{j}^{\,q} = -K \nabla \vec{T}$$

K ↑ thermal conductivity, and is +ve

since thermal current flows opposite to the direction of the temp. gradient



e^- arriving at the centre of the bar from the left had their last collision in the high temp. region. Those arriving at the centre from right had last collision in the low temp. region. Hence electrons moving to the right at the centre of the bar tend to more energetic than those moving to the left yielding a net thermal current to the right

Tak in 1-d case — e^- can move only in x-direction.

So at point x — half of the e^- come from high T side and half from low.

If $\mathcal{E}(T) \to$ thermal energy per e^- in a metal at eq^m at temp. T.

then an e^- whose last collision was at x' will, on the average, have a thermal energy $\mathcal{E}(T[x'])$. The electrons arriving at x from the high temp. side will, on the average have had their last collisions at $x - u\tau$, and will therefore carry a thermal energy per e^- of size $\mathcal{E}(T[x-u\tau])$. Their contribution to the thermal current density at x will therefore be a number of such e^-'s per unit vol., $n/2$, times their velocity, u, times this energy,

or $\frac{n}{2} u \, \mathcal{E}(T[x-u\tau])$

e^- arriving at x from low temp side will contribute

$\frac{n}{2}(-u)\{\mathcal{E}(T[x+u\tau])\}$

$$\therefore \; J^q = \frac{1}{2} n u \left[\mathcal{E}(T[x-u\tau]) - \mathcal{E}(T[x+u\tau])\right]$$

$$J^q = n u^2 \tau \frac{d\mathcal{E}}{dT}\left(\frac{-dT}{dx}\right)$$

for 3-d case

$$\langle v_x^2 \rangle = \langle v_y^2 \rangle = \langle v_z^2 \rangle = \frac{1}{3} u^2$$

$$\because n \frac{d\mathcal{E}}{dT} = \frac{N}{V} \frac{d\mathcal{E}}{dT} = \frac{1}{V} \frac{dE}{dT} = C_V$$

$$\vec{j}^{\,q} = \frac{1}{3} u^2 \tau C_V (-\vec{\nabla} T)$$

$$K = \frac{1}{3} u^2 \tau C_V$$

$$\boxed{K = \frac{1}{3} l u C_V}$$

$$\frac{K}{\sigma} = \frac{\frac{1}{3} C_V m u^2}{n e^2}$$

Drude took, $C_V = \frac{3}{2} n k_B$ from classical Ideal gas law

$$\frac{1}{2} m u^2 = \frac{3}{2} k_B T$$

$$\frac{K}{\sigma} = \frac{3}{2} \left(\frac{k_B}{e}\right)^2 T$$

$$\boxed{\frac{K}{\sigma T} = \frac{3}{2} \left(\frac{k_B}{e}\right)^2 = 1.11 \times 10^{-8} \text{ watt-ohm}/K^2}$$

no electronic contribution to the specific heat remotely comparable to $3/2\, n k_B$ was ever observed

Two errors from Drude

At room temp. the actual electronic contribution to specific heat is about 100 times smaller than the classical prediction.

→ mean electronic speed is is about 100 times larger